LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

17/18UMT3MC01 – INTEGRAL TRANSFORMS AND PARTIAL DIFF. EQUATIONS

Date: 24-11-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions **(10 x 2 = 20 Marks)**

1. Define Laplace transform.
2. What is $L\{t^3\}$?
3. Find $L^{-1}\left\{\frac{1}{(s-3)^2}\right\}$.
4. Find $L^{-1}\left\{\frac{1}{(s^2+k^2)}\right\}$, where k is a constant.
5. Define Fourier transform.
6. Prove that $F\{af(x)+bg(x)\} = aF\{f(x)\} + bF\{g(x)\}$.
7. Define Fourier sine transform.
8. Prove that $F_c\{f(ax)\} = \frac{1}{a}F_c\left(\frac{s}{a}\right)$.
9. Eliminate the arbitrary constants and form a differential equation from $z = ax + by$.
10. Solve $\frac{\partial^2 z}{\partial y^2} = \sin y$.

PART – B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11. Find (i) $L\{t\, sin^2 t\}$ and (ii) $L\{t^2 \cos 4t\}$.
12. Find the Laplace transform of the rectangular wave given by $f(t) = \begin{cases} 1, & for\ 0 < t \le b \\ -1, & for\ b < t < 2b \end{cases}$.
13. Evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$.
14. Find $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$.
15. Prove that $F\left\{\frac{d^n}{dx^n}(f(x))\right\} = (-is)^n F(s)$, provided $f(x), f'(x), f''(x), \dots, f^{(n-1)}x \to 0$ as $x \to \pm\infty$.
16. Prove that $F_c\left\{\frac{1}{\sqrt{x}}\right\} = F_s\left\{\frac{1}{\sqrt{x}}\right\} = \frac{1}{\sqrt{s}}$.
17. Solve $p^2 + q^2 = x^2 + y^2$.
18. Solve $\sqrt{p} + \sqrt{q} = 1$.

PART – C

Answer any TWO questions **(2 x 20 = 40 Marks)**

19. (a). Using Laplace transform evaluate $\int_0^\infty te^{-3t}\cos t\ dt$.

(b). Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\}$.

(10+10)

20. Using Laplace transform solve the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$, given that $y=\frac{dy}{dt}=0$ when $t=0$. **(20)**

21. (a). State and prove the convolution theorem on Fourier transform.

(b). State and prove the Parseval's identity.

(10+10)

22. (a). Find the complete solution of $p(1+q^2)=q(z-1)$.

(b). Solve $(y-z)p+(z-x)q=x-y$.

(10+10)

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